## *Note on the Motion about an Attracting Centre of slowly increasing Mass.* By H. C. Plummer, M.A.

1. The dynamical problem of motion under the attraction of a body of increasing mass was suggested by Oppolzer * in an attempt to explain a part of the secular acceleration of the Moon as a result of the deposition of meteoric matter on the Earth. It has since been discussed by Gyldén,† Mestschersky,‡ Lehmann-Filhés,§ and E. Strömgren.‖

The same theory is recalled by Mr. Cowell's conclusion that the motion of the Earth round the Sun is also affected by a secular acceleration. It is true that the idea that this can be attributed to a gradual increase in the mass of the Sun is not borne out by Mr. Cowell's result for the motion of *Mercury* ; but this consideration may not be quite decisive, since the motion of this planet is known to be incompletely explained by theory. However this may be, the effect of an increase in the central body is a point of some interest, and a short discussion is here given of the simplest case, that in which the rate of increase is constant.

2. Let $\mu u^2(1+\alpha t)$ represent the law of central attraction, $u$ being the reciprocal of the radius vector. Since the force is central, the integral of areas holds, and the equations of motion may be written

$$r^2\frac{d\theta}{dt}=h \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

$$\frac{d^2u}{d\theta^2}+u=\frac{\mu}{h^2}(1+\alpha t) \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

Now if powers of $\alpha$ above the first be neglected, it is easily verified that these equations are satisfied by the solution

$$u=\frac{\mu}{h^2}\{1+e\cos(\theta-\gamma)\}(1+\alpha t) \quad \dots \quad \dots \quad (3)$$

where

$$u=\frac{\mu}{h^2}\{1+e\cos(\theta-\gamma)\} \quad \dots \quad \dots \quad \dots \quad (4)$$

can be regarded as the undisturbed (though not the osculating) orbit at the time $t=0$. Let R and T be the radius vector and time in this undisturbed orbit corresponding to the anomaly $\theta$. Then in the disturbed orbit

$$r=\mathrm{R}/(1+\alpha t)$$

and

$$\int \mathrm{R}^2 d\theta = h\int(1+\alpha t)^2\,dt$$

or

$$\mathrm{T}=t(1+\alpha t)$$

* *A. N.*, 2573. † *A. N.*, 2593. ‡ *A. N.*, 3153, 3807.
§ *A. N.*, 3480. ‖ *A. N.*, 3897.

Hence at any time the deviations of the disturbed from the undisturbed orbit for the same anomaly $\theta$ are given by

$$\frac{\Delta r}{r} = \frac{\Delta t}{t} = -\alpha t \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

and the increase in the anomaly in a given time $t$ is

$$-\frac{d\theta}{dt}\Delta t = \alpha h r^{-2} t^2 \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

3. The required results are thus obtained more simply and directly than by the variation of elements; but the first-order variations of $e$, $\gamma$ and $a$ can be deduced very easily. In the equation (3) of the actual trajectory, $t$ is to be regarded as a function of $\theta$; the osculating orbit at the point $\theta$ is

$$u = \frac{\mu}{h^2}\{1 + e' \cos(\theta - \gamma')\}(1 + \alpha t) \quad \dots \quad \dots \quad (7)$$

where $t$ is to be regarded as constant. The identity of the radius and tangent in the two curves gives

$$e' \cos(\theta - \gamma') = e \cos(\theta - \gamma)$$
$$e' \sin(\theta - \gamma') = e \sin(\theta - \gamma) - \alpha h r/\mu$$

which lead to

$$\Delta(e \cos \gamma) = -\alpha h r \sin \theta/\mu$$
$$\Delta(e \sin \gamma) = \alpha h r \cos \theta/\mu$$

or finally to

$$\Delta e = -\frac{\alpha h}{\mu} r \sin(\theta - \gamma) = -\alpha \cdot \frac{h^3}{\mu^2} \cdot \frac{\sin \mathrm{E}}{(1 - e^2)^{\frac{1}{2}}} \quad \dots \quad (8)$$

$$\Delta \gamma = \frac{\alpha h}{e\mu} r \cos(\theta - \gamma) = \alpha \cdot \frac{h^3}{\mu^2} \cdot \frac{\cos \mathrm{E} - e}{e(1 - e^2)} \quad \dots \quad (9)$$

where E is the eccentric anomaly corresponding to $\theta$.

The mean distance in the osculating orbit is

$$a' = \frac{h^2}{\mu(1 - e'^2)(1 + \alpha t)}$$

Hence

$$\Delta a = a\left(-\alpha t + \frac{2e\Delta e}{1 - e^2}\right)$$

$$= -\alpha a\left\{t + \frac{2h^3}{\mu^2} \cdot \frac{e \sin \mathrm{E}}{(1 - e^2)^{\frac{3}{2}}}\right\} \quad \dots \quad \dots \quad (10)$$

Equations (8), (9) and (10) may be compared with equations (25) of Dr. Strömgren's paper; the two sets differ by additive constants which can be introduced into the above to annul the variations at a given epoch, as $t = 0$. The variations found above are referred to the intermediate orbit whose equation is (4).

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*Mean Areas and Heliographic Latitudes of Sun-spots in the Year 1904, deduced from Photographs taken at the Royal Observatory, Greenwich; at Dehra Dûn; at Kodaikánal Observatory, India; and in Mauritius.*

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxv. p. 151, and are deduced from the measurements of photographs taken at the Royal Observatory, Greenwich; at Dehra Dûn; at the Kodaikánal Observatory, India; and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in 1904; and Table II. gives the same particulars for the entire year 1904 and the three preceding years for the sake of comparison. The areas are given in two forms: first, projected areas; that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in 1904 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted area for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots; and Table IV. gives the same information for the year as a whole, similar results for the three preceding years being added, as in the case of Table II.

Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*, and for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1903 on pp. 152 and 153 of vol. lxv.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

The principal features of the record for 1904 are:

1. The comparatively slow, though steady, increase in the mean daily spotted area, the umbræ showing an advance on 1903 of only 32 per cent.; the whole spots of 44 per cent. The years in the two preceding cycles showing this relation to the years which they followed were 1882 and 1893, when the actual maximum was very close at hand.